Chapter 13 - Inventory Management

3. \( D = 1,215 \) bags/yr.
   \( S = $10 \)
   \( H = $75 \)
   a. \( Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1,215)(10)}{75}} = 18 \) bags
   b. \( Q/2 = 18/2 = 9 \) bags
   c. \( \frac{D}{Q} = \frac{1,215 \text{ bags}}{18 \text{ bags/\text{orders}}} = 67.5 \text{ orders} \)
   d. \( TC = \frac{Q}{2}H + \frac{DS}{Q} \)
      \[ \frac{18}{2}(75) + \frac{1,215}{18}(10) = 675 + 714.71 = $1,350 \]
   e. Assuming that holding cost per bag increases by $9/bag/year
      \( Q = \sqrt{\frac{2(1,215)(10)}{84}} = 17 \) bags
      \( TC = \frac{17}{2}(84) + \frac{1,215}{17}(10) = 714 + 714.71 = $1,428.71 \)
      Increase by \([$1,428.71 - $1,350] = $78.71\)

4. \( D = 40/\text{day} \times 260 \text{ days/yr.} = 10,400 \) packages
   \( S = $60 \)
   \( H = $30 \)
   a. \( Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(10,400)(60)}{30}} = 203.96 = 204 \) boxes
   b. \( TC = \frac{Q}{2}H + \frac{DS}{Q} \)
      \[ \frac{204}{2}(30) + \frac{10,400}{204}(60) = 3,060 + 3,058.82 = $6,118.82 \]
   c. Yes
   d. \( TC_{200} = \frac{200}{2}(30) + \frac{10,400}{200}(60) \)
      \[ TC_{200} = 3,000 + 3,120 = $6,120 \]
      \[ 6,120 - 6,118.82 \] (only $1.18 higher than with EOQ, so 200 is acceptable.)
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8. D = 27,000 jars/month
H = $.18/month
S = $60

a. \( Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(27,000)60}{.18}} = 4,242.64 \approx 4,243. \)

\( TC = \frac{Q}{2}H + \frac{D}{Q}S \)

\( TC_{4,000} = $765.00 \)

\( TC_{4,243} = $736.67 \)

\( TC_{4,243} - TC_{4,000} = $1.32 \) Difference

\( TC_{4000} = \left( \frac{4,000}{2} \right) (.18) + \left( \frac{27,000}{4,000} \right) (60) = 765 \)

\( TC_{4243} = \left( \frac{4,243}{2} \right) (.18) + \left( \frac{27,000}{4,243} \right) (60) = 763.68 \)

b. Current: \( \frac{D}{Q} = \frac{27,000}{4,000} = 6.75 \)

\( Q = \sqrt{\frac{2DS}{H}} \) So 2,700 = \( \sqrt{\frac{2(27,000)S}{.18}} \)

Solving, \( S = $24.30 \)

c. The carrying cost happened to increase rather dramatically from $.18 to approximately $.3705.

\( Q = \sqrt{\frac{2DS}{H}} = 2,700 = \sqrt{\frac{2(27,000)S}{.18}} \)

Solving, \( H = $.3705 \)
14. a. $S = 48$

\[
D = 25 \text{ stones/day} \times 200 \text{ days/yr.} = 5,000 \text{ stones/yr.}
\]

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Unit Price</th>
<th>a. $H = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – 399</td>
<td>$10</td>
<td></td>
</tr>
<tr>
<td>400 – 599</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>600 +</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

\[
Q = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(5,000)(48)}{2}} = 489.90
\]

\[
TC_{490} = \frac{490}{2} \times 2 + \frac{5,000}{490} \times 48 + 9(5,000) = 45,980
\]

\[
TC_{600} = \frac{600}{2} \times 2 + \frac{5,000}{600} \times 48 + 8(5,000) = 41,000
\]

\[
\therefore 600 \text{ is optimum.}
\]

b. $H = .30P$

\[
EOQ_{88} = \sqrt{\frac{2(5,000)(48)}{.30(8)}} = 447 \text{ NF}
\]

(Not feasible at $8/stone)

\[
EOQ_{89} = \sqrt{\frac{2(5,000)(48)}{.30(9)}} = 422
\]

(Feasible)

Compare total costs of the EOQ at $9 and lower curve’s price break:

\[
TC = \frac{Q}{2} (.30P) + \frac{D}{Q} (S) + PD
\]

\[
TC_{422} = \frac{422}{2} (.30(9)) + \frac{5,000}{422} (\$48) + 9(5,000) = 46,139
\]

\[
TC_{600} = \frac{600}{2} (.30(8)) + \frac{5,000}{600} (\$48) + 8(5,000) = 41,120
\]

\[
\text{Since an order quantity of 600 would have a lower cost than 422, 600 stones is the optimum order size.}
\]

c. ROP = 25 stones/day (6 days) = 150 stones.
15. 

<table>
<thead>
<tr>
<th>Range</th>
<th>P</th>
<th>H</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–999</td>
<td>$5.00</td>
<td>$2.00</td>
<td>495</td>
</tr>
<tr>
<td>1,000–3,999</td>
<td>4.95</td>
<td>1.98</td>
<td>497 NF</td>
</tr>
<tr>
<td>4,000–5,999</td>
<td>4.90</td>
<td>1.96</td>
<td>500 NF</td>
</tr>
<tr>
<td>6,000+</td>
<td>4.85</td>
<td>1.94</td>
<td>503 NF</td>
</tr>
</tbody>
</table>

D = 4,900 seats/yr.
H = .4P
S = $50

Compare TC_{495} with TC for all lower price breaks:

TC_{495} = \frac{495}{2} (-2) + \frac{4,900}{495} (50) + 5.00(4,900) = 25,490

TC_{1,000} = \frac{1,000}{2} (1.98) + \frac{4,900}{1,000} (50) + 4.95(4,900) = 25,490

TC_{4,000} = \frac{4,000}{2} (1.96) + \frac{4,900}{4,000} (50) + 4.90(4,900) = 27,991

TC_{6,000} = \frac{6,000}{2} (1.94) + \frac{4,900}{6,000} (50) + 4.85(4,900) = 29,626

Hence, one would be indifferent between 495 or 1,000 units.
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17. D = 3600 boxes per year

\[ Q = 800 \text{ boxes (recommended)} \]
\[ S = $80 \text{ /order} \]
\[ H = $10 \text{ /order} \]

If the firm decides to order 800, the total cost is computed as follows:

\[ TC = \left( \frac{Q}{2} \right) H + \left( \frac{D}{Q} \right) S + (P \times D) \]

\[ TC_{Q=800} = \left( \frac{800}{2} \right) \times 10 + \left( \frac{3600}{800} \right) \times 80 + (3,600 \times 1.1) \]

\[ TC_{Q=800} = 4,000 + 360 + 3,960 = 8,320 \]

Even though the inventory total cost curve is fairly flat around its minimum, when there are quantity discounts, there are multiple U shaped total inventory cost curves for each unit price depending on the unit price. Therefore when the quantity changes from 800 to 801, we shift to a different total cost curve.

If we take advantage of the quantity discount and order 801 units, the total cost is computed as follows:

\[ TC = \left( \frac{Q}{2} \right) H + \left( \frac{D}{Q} \right) S + (P \times D) \]

\[ TC_{Q=801} = \left( \frac{801}{2} \right) \times 10 + \left( \frac{3600}{801} \right) \times 80 + (3,600 \times 1.0) \]

\[ TC_{Q=801} = 4,005 + 359.55 + 3,600 = 7,964.55 \]

The order quantity of 801 is preferred to order quantity of 800 because \( TC_{Q=801} < TC_{Q=800} \) or 7964.55 < 8320.

\[ EOQ = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(3600)(80)}{10}} = 240 \text{ boxes} \]

\[ TC_{EOQ} = \left( \frac{Q}{2} \right) H + \left( \frac{D}{Q} \right) S + (P \times D) \]

\[ TC_{EOQ} = \left( \frac{240}{2} \right) \times 10 + \left( \frac{3600}{240} \right) \times 80 + (3,600 \times 1.1) \]

\[ TC_{EOQ} = 1,200 + 1,200 + 3,960 = 6,360 \]

The order quantity of 800 is not around the flat portion of the curve because the optimal order quantity (EOQ) is much lower than the suggested order quantity of 800. Since the EOQ of 240 boxes provides the lowest total cost, it is the recommended order size.
24. SL ≥ 96% → Z = 1.75
\[\overline{d} = 12 \text{ units/day} \quad LT = 4 \text{ days} \]
\[\sigma_d = 2 \text{ units/day} \quad \sigma_{LT} = 1 \text{ day} \]
\[\text{ROP} = \overline{d}LT + Z\sqrt{LT\sigma_d^2 + \overline{d}^2\sigma_{LT}^2} = 12(4) + 1.75\sqrt{4(4) + 144(1)} \]
\[= 48 + 1.75(12.65) = 50 + 22.14 = 72.14 \]

26. \[d = 5 \text{ boxes/wk.} \quad \sigma_d = .5 \text{ boxes/wk.} \]
\[LT = 2 \text{ wk.} \]
\[S = $2 \quad H = $.20/box \]
a. \[Q_0 = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(260)}{.20}} = 72.11 \text{ [round to 72]} \]
b. \[\text{ROP} = \overline{d}LT + z\sqrt{LT\sigma_d} \]
\[z = \frac{\text{ROP} - \overline{d}(LT)}{\sqrt{LT}\sigma_d} = \frac{12 - 5(2)}{\sqrt{2}(.5)} = 2.83 \]
Area under curve to left is .9977, so risk = 1.0000 - .9977 = .0023

c. \[Q_0 = \overline{d}(O + LT) + z\sigma_d\sqrt{O + LT - A} \]
Thus,
\[36 = 5(7 + 2) + z(.5)\sqrt{7 + 2} - 12 \]
Solving for \(z\) yields \(z = +2.00\) which implies a risk of \(1.000 - .9772 = .0228\).

36. \[C_s = \text{Rev} - \text{Cost} = $5.70 - $4.20 = $1.50/\text{unit} \quad \overline{d} = 80 \text{ lb./day} \]
\[C_c = \text{Cost - Salvage} = $4.20 - $2.40 = $1.80/\text{unit} \quad \sigma_d = 10 \text{ lb./day} \]
\[\text{SL} = \frac{C_s}{C_s + C_c} = \frac{$1.50}{$1.50 + $1.80} = 0.4545 \]
The corresponding \(z = -0.11\)
\[S_o = \overline{d} + z\sigma_d = 80 - 0.11(10) = 78.9 \text{ lb.} \]
37. \( \overline{d} = 40 \text{ qt./day} \)  A stocking level of 49 quarts translates into a \( z \) of \(+1.5\):

s.d. of demand = 6 qt./day

\[ C_e = \$0.35/\text{qt} \]

\[ z = \frac{S - \overline{d}}{\sigma_d} = \frac{49 - 40}{6} = 1.5 \]

\[ S = 49 \text{ qt.} \]  This implies a service level of \( .9332 \):

SL = \( \frac{C_s}{C_s + C_e} \)  Thus, \( .9332 = \frac{C_s}{C_s + \$0.35} \)

Solving for \( C_s \) we find: \( .9332(C_s + \$0.35) = C_s \); \( C_s = \$4.89/\text{qt.} \)

Customers may buy other items along with the strawberries (ice cream, whipped cream, etc.) that they wouldn’t buy without the berries.