Chapter 10 - Quality Control

**Solutions**

1. specs: 24 oz. to 25 oz.
   \[ \mu = 24.5 \text{ oz. } [\text{assume } \mu = \bar{x}] \]
   \[ \sigma = .2 \text{ oz.} \]
   a. [refers to population]
   \[ z = \frac{+.5}{.2} = 2.5 \rightarrow 2(0.0062) = .0124 \]
   b. \[ \mu \pm 2 \frac{\sigma}{\sqrt{n}} = 24.5 \pm 2 \frac{.2}{\sqrt{16}} = 24.5 \pm .10 \text{ or } 24.40 \text{ to } 24.60 \]

2. \[ \mu = 1.0 \text{ liter} \]
   \[ \sigma = .01 \text{ liter} \]
   \[ n = 25 \]
   a. Control limits: \[ \mu \pm z \frac{\sigma}{\sqrt{n}} \]
   \[ [z = 2.17 \text{ for } 97\%] \]
   UCL is \[ 1.0 + 2.17 \frac{.01}{\sqrt{25}} = 1.043 \]
   LCL is \[ 1.0 - 2.17 \frac{.01}{\sqrt{25}} = .9957 \]

3. a. \[ n = 20 \]
   \[ A_2 = 0.18 \]
   \[ \bar{X} = 3.10 \text{ Mean Chart: } \bar{X} \pm A_2 \bar{R} = 3.1 \pm 0.18(0.45) \]
   \[ D_3 = 0.41 \]
   \[ \bar{R} = 0.45 \]
   \[ D_4 = 1.59 \]
   Hence, UCL is 3.181 and LCL is 3.019. All means are within these limits.

   Range Chart: UCL is \[ D_4 \bar{R} = 1.59(0.45) = 0.7155 \]
   LCL is \[ D_3 \bar{R} = 0.41(0.45) = .1845 \]

   In control since all points are within these limits.

4. | Sample | Mean | Range |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79.48</td>
<td>2.6</td>
</tr>
<tr>
<td>2</td>
<td>80.14</td>
<td>2.3</td>
</tr>
<tr>
<td>3</td>
<td>80.14</td>
<td>1.2</td>
</tr>
<tr>
<td>4</td>
<td>79.60</td>
<td>1.7</td>
</tr>
<tr>
<td>5</td>
<td>80.02</td>
<td>2.0</td>
</tr>
<tr>
<td>6</td>
<td>80.38</td>
<td>1.4</td>
</tr>
</tbody>
</table>

   [Both charts suggest the process is in control: Neither has any points outside the limits.]
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5. n = 200
   a. \[1\quad 2\quad 3\quad 4\]
      \[.020\quad .010\quad .025\quad .045\]
   b. \[2.0 + 1.0 + 2.5 + 4.5)/4 = 2.5\%
   c. mean = .025
      \[\text{Std. dev.} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{.025(.975)}{200}} = .011\]
   d. \[z = 2.17\]
      \[.025 \pm 2.17(.011) = .025 \pm .0239 = .0011 \text{ to } .0489.\]
   e. \[.025 + z(.011) = .047\]
      Solving, \[z = 2,\] leaving \[.0228\] in each tail. Hence, \[\alpha = 2(.0228) = .0456.\]
   f. Yes.
   g. mean = .02
      \[\text{Std. dev.} = \sqrt{\frac{.02(.98)}{200}} = .0099 \text{ [round to .01]}\]
   h. \[.02 \pm 2(.01) = 0 \text{ to } .04\]
      The last sample is beyond the upper limit.

7. \[\bar{c} = \frac{110}{14} = 7.857\]
   Control limits: \[\bar{c} \pm 3\sqrt{\bar{c}} = 7.857 \pm 8.409\]
   UCL is 16.266, LCL becomes 0.
   All values are within the limits.

9. \[\bar{p} = \frac{\text{total number of defectives}}{\text{total number of observations}} = \frac{87}{16(100)} = .054\]
   Control limits are \[\bar{p} \pm z\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = .054 \pm 1.96\sqrt{\frac{.054(.946)}{100}} = .054 \pm .044.\] Hence, UCL = 0.10
   LCL = 0.01
   Note that observations must be converted to fraction defective, or control limits must be converted to number of defectives. In the latter case, the upper control limit would be 10 defectives and the lower control limit would be 1 defective.

13. a.
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14. a. [Data from Chapter 10, Problem 8]
Median is 1.5  A = Above, B = Below, U = Up, D = Down.
Sample:  1  2  3  4  5  6  7  8  9  10  11  12  13  14
Median:  A A B B B A A B A B A B A B A
Data:  2  3  1  0  1  3  2  0  2  1  3  1  2  0
Up/down: – U D D U U D D U D U D U D

b. [Data from #7] Median is 7.5.
Day:  1  2  3  4  5  6  7  8  9  10  11  12  13  14
Median:  B A A A A B B A A B B B B A
Data:  4  10  14  8  9  6  5  12  13  7  6  4  2  10
Up/down: – U U D U D U D U D U D U

For part a and b:

\[ E(r)_{med} = \frac{N}{2} + 1 = \frac{14}{2} + 1 = 8 \text{ runs} \]
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\[
E(r)_{u/d} = \frac{2N - 1}{3} = \frac{2(14) - 1}{3} = 9 \text{ runs}
\]

\[
\sigma_{\text{med}} = \sqrt{\frac{N - 1}{4}} = \sqrt{\frac{14 - 1}{4}} = 1.803 \text{ runs}
\]

\[
\sigma_{u/d} = \sqrt{\frac{16N - 29}{90}} = \sqrt{\frac{224 - 29}{90}} = 1.472 \text{ runs}
\]

For part a:

\[
Z_{\text{med}} = \frac{10 - 8}{1.803} = 1.09 \quad Z_{\text{up/down}} = \frac{10 - 9}{1.472} = .679
\]

For part b:

\[
Z_{\text{med}} = \frac{6 - 8}{1.803} = -1.109 \quad Z_{\text{up/down}} = \frac{7 - 9}{1.472} = -1.36
\]

Since the absolute values of all Z statistics calculated above are less than 2, all patterns appear to be random.

Summary:

<table>
<thead>
<tr>
<th>Test</th>
<th>obs.</th>
<th>Exp.</th>
<th>( \sigma )</th>
<th>( z )</th>
<th>Conclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. median</td>
<td>10</td>
<td>8</td>
<td>1.80</td>
<td>1.11</td>
<td>random</td>
</tr>
<tr>
<td>up/down</td>
<td>10</td>
<td>9</td>
<td>1.47</td>
<td>.68</td>
<td>random</td>
</tr>
<tr>
<td>b. median</td>
<td>6</td>
<td>8</td>
<td>1.80</td>
<td>-1.11</td>
<td>random</td>
</tr>
<tr>
<td>up/down</td>
<td>7</td>
<td>9</td>
<td>1.47</td>
<td>-1.36</td>
<td>random</td>
</tr>
</tbody>
</table>

22. 

<table>
<thead>
<tr>
<th>Machine</th>
<th>Standard Deviation (in.)</th>
<th>Job Specification (±in.)</th>
<th>( C_p )</th>
<th>Capable ?</th>
</tr>
</thead>
<tbody>
<tr>
<td>001</td>
<td>0.02</td>
<td>0.05</td>
<td>0.833</td>
<td>No</td>
</tr>
<tr>
<td>002</td>
<td>0.04</td>
<td>0.07</td>
<td>0.583</td>
<td>No</td>
</tr>
<tr>
<td>003</td>
<td>0.10</td>
<td>0.18</td>
<td>0.600</td>
<td>No</td>
</tr>
<tr>
<td>004</td>
<td>0.05</td>
<td>0.15</td>
<td>1.000</td>
<td>No/yes</td>
</tr>
<tr>
<td>005</td>
<td>0.01</td>
<td>0.04</td>
<td>1.333</td>
<td>Yes</td>
</tr>
</tbody>
</table>

24. 

Let USL = Upper Specification Limit, LSL = Lower Specification Limit, \( \bar{X} \) = Process mean, \( \sigma \) = Process standard deviation

For process H:
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\[
\frac{\bar{X} - \text{LSL}}{3\sigma} = \frac{15 - 14.1}{3 \cdot (0.32)} = 0.93
\]

\[
\frac{\text{USL} - \bar{X}}{3\sigma} = \frac{16 - 15}{3 \cdot (0.32)} = 1.04
\]

\[C_{pk} = \min\{0.93, 1.04\} = 0.93\]

\[0.93 < 1.0, \text{ not capable}\]

For process K:

\[
\frac{\bar{X} - \text{LSL}}{3\sigma} = \frac{33 - 30}{3 \cdot (1)} = 1.0
\]

\[
\frac{\text{USL} - \bar{X}}{3\sigma} = \frac{36.5 - 33}{3 \cdot (1)} = 1.17
\]

\[C_{pk} = \min\{1.0, 1.17\} = 1.0\]

Assuming the minimum acceptable \(C_{pk}\) is 1.33, since 1.0 < 1.33, the process is not capable.

For process T:

\[
\frac{\bar{X} - \text{LSL}}{3\sigma} = \frac{18.5 - 16.5}{3 \cdot (0.4)} = 1.67
\]

\[
\frac{\text{USL} - \bar{X}}{3\sigma} = \frac{20.1 - 18.5}{3 \cdot (0.4)} = 1.33
\]

\[C_{pk} = \min\{1.67, 1.33\} = 1.33\]

Since 1.33 = 1.33, the process is capable.