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Inventory Management

Homework problems # 3,4,8, 14,15,17, 24,26,36,37 on pp. 589-595.

Inventory

• A stock or store of goods
• Independent demand items
  • Items that are ready to be sold or used

Independent vs. Dependent Demand

Independent Demand

• When item’s demand is influenced by market conditions and is not related to (i.e., is “independent” of) production decision for any other item.
• Wholesale and retail merchandise (finished goods), service industry inventory, end-item and replacement-part inventories, spare-parts, MRO (maintenance, repair, and operating) supplies.
• Demand must be forecasted.

Dependent Demand

• When item’s demand derives from (i.e., “depend” on) the production decisions for its parents.
• All intermediate and purchased items in manufacturing.
• Demand must be derived.

Types of Inventories

• Raw materials and purchased parts
• Work-in-process
• Finished goods inventories or merchandise
• Maintenance and repairs (MRO) inventory, tools and supplies
• Goods-in-transit to warehouses or customers (pipeline inventory) e.g., Dell

Functions of Inventory

• Inventories serve a number of functions such as:
  1. To meet anticipated customer demand
  2. To smooth production requirements
  3. To decouple operations
  4. To protect against stockouts
  5. To take advantage of order cycles
  6. To hedge against price increases
  7. To permit operations
  8. To take advantage of quantity discounts
Management has two basic functions concerning inventory:
1. Establish a system for tracking items in inventory
2. Make decisions about
   • How much to order?
   • When to order?

A system to keep track of inventory
A reliable forecast of demand
Knowledge of lead times
Reasonable estimates of
   • Holding costs
   • Ordering costs
   • Shortage costs
A classification system

Physical count of items in inventory made at periodic intervals
System that keeps track of removals from inventory continuously, thus monitoring current levels of each item
Two containers of inventory; reorder when the first is empty

Forecasts
   Inventories are necessary to satisfy customer demands, so it is important to have a reliable estimates of the amount and timing of demand
Lead time
   Time interval between ordering and receiving the order
Point-of-sale (POS) systems
   A system that electronically records actual sales
   Such demand information is very useful for enhancing forecasting and inventory management

Classifying inventory according to some measure of importance, and allocating control efforts accordingly
10 to 20 percent of the number of items in inventory and about 60 to 70 percent of the annual dollar value
Moderately important
50 to 60 percent of the number of items in inventory but only about 10 to 15 percent of the annual dollar value

A physical count of items in inventory
Cycle counting management
   How much accuracy is needed?
   • A items: ± 0.2 percent
   • B items: ± 1 percent
   • C items: ± 5 percent
   When should cycle counting be performed?
   Who should do it?
Inventory Management

How Much to Order: EOQ Models

• Economic order quantity model
• Economic production model
• Quantity discount model

Basic EOQ Model

• The basic EOQ model is used to find a fixed order quantity that will minimize total annual inventory costs

Assumptions

• Only one product is involved (at a time)
• Annual demand requirements are known
• Demand is even throughout the year
• Lead time does not vary
• Each order is received in a single delivery (no partial delivery)
• There are no quantity discounts

Key Inventory Terms

• Lead time: time interval between ordering and receiving the order
• Holding (carrying) costs: cost to carry an item in inventory for a length of time, usually a year
• Ordering costs: costs of ordering and receiving inventory
• Shortage costs: costs when demand exceeds supply

The Inventory Cycle

Profile of Inventory Level Over Time

When order quantity is Q, how much is average inventory?

Large vs. Small orders

Total Cost

Total cost = Annual carrying cost + Annual ordering cost

\[ TC = \frac{Q}{2} H + \frac{D}{Q} S \]
Inventory Management

Cost Minimization Goal

The Total-Cost Curve is U-Shaped

Order	Quantity	(Q)
The	Total	Cost	Curve	is	U-Shaped
Ordering Costs	\( \frac{Q}{2} \times D \)
Holding costs	\( \frac{Q}{2} \times H \)
Annual Cost

How to find the optimal order quantity?

Deriving the EOQ

Using calculus, we take the derivative of the total cost function and set the derivative (slope) equal to zero and solve for Q.

\[
Q_{opt} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(\text{Annual Demand})(\text{Order or Setup Cost})}{\text{Annual Holding Cost}}}
\]

Minimum Total Cost

The total cost curve reaches its minimum where the carrying and ordering costs are equal.

EOQ Sensitivity

- What happens to optimal order quantity (and cycle inventory) if the demand rate increase?
- What happens to lot sizes if setup/ordering costs decrease?
- What happens if interest rates drop?
- How critical are errors in estimating D, H, and S?
- Conclusion: EOQ is robust, insensitive to parameter estimation errors!

EOQ Sensitivity

Production done in batches or lots
Capacity to produce a part exceeds the part’s usage or demand rate
Assumptions of EPQ are similar to EOQ except orders are received incrementally during production

Economic Production Quantity (EPQ)

Skip EPQ.
EOQ with Quantity Discounts

- Objectives:
  - Consider tradeoffs between discount benefits and increases in holding costs
  - The introduction of quantity discounts will cause the optimum order quantity to be unchanged or greater.

- Types of Discounts:
  - All-units:
  - Increment:

Total Costs with Quantity Discounts

\[
TC = \frac{Q^2}{2H} + \frac{DS}{Q} + PD
\]

Total Costs with constant PD

Total Costs with Variable PD

EOQ with Quantity Discounts

- Two-Step Procedure
  - Step 1. Beginning with lowest price, calculate the EOQ for each price level until a feasible EOQ is found. It is feasible if it lies in the range corresponding to its price.
  - Step 2. If the first feasible EOQ found is for the lowest price level, this quantity is best. Otherwise, calculate the total cost for the first feasible EOQ and for the price break quantity at each lower price level. The quantity with the lowest total cost is optimal.

Quantity Discounts Example

Example. Assume that the following quantity discount schedule is provided by a supplier:

<table>
<thead>
<tr>
<th>Order size</th>
<th>Discount</th>
<th>Unit cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0~49</td>
<td>0%</td>
<td>$30.00</td>
</tr>
<tr>
<td>50~99</td>
<td>5%</td>
<td>$28.50</td>
</tr>
<tr>
<td>100 or more</td>
<td>10%</td>
<td>$27.00</td>
</tr>
</tbody>
</table>

If annual demand is 120 units, ordering cost is $20 per order, and annual unit inventory holding cost is 25%, what order quantity would minimize total inventory costs?
When to Reorder with EOQ Ordering

- **Reorder Point** - When the quantity on hand of an item drops to this amount, the item is reordered.
- **Safety Stock** - Stock that is held in excess of expected demand due to variable demand rate and/or lead time.
- **Service Level** - Probability that demand will not exceed supply during lead time.

Determinants of the Reorder Point

- The rate of demand
- The lead time
- Demand and/or lead time variability
- Stockout risk (safety stock)

Example:

Reorder Point Models

- Variable demand and constant lead time
  \[ \text{ROP} = \bar{d} \cdot LT + Z \cdot \sigma_d \cdot \sqrt{LT} \]
- Constant demand and variable lead time
  \[ \text{ROP} = \bar{d} \cdot LT + Z \cdot \bar{d} \cdot \sigma_{LT} \]
- Variable demand and variable lead time
  \[ \text{ROP} = \bar{d} \cdot LT + Z \cdot \sqrt{LT \cdot \sigma_d^2 + \bar{d}^2 \cdot \sigma_{LT}^2} \]

Where: \( \bar{d} \) = average daily or weekly demand,
\( \sigma_d \) = standard deviation of demand per day or week,
\( \sigma_{LT} \) = standard deviation of lead time per day or week.
Reorder Point Example

The injection molding department of a company uses 40 pounds of a powder a day. Inventory is reordered when the amount on hand is 240 pounds. Lead time averages five days. It is normally distributed and has a standard deviation of two days.

a). What is the probability of a stockout during lead time?
b). What reorder point would provide a 5% stockout?

Fixed-order-interval (FOI) model

- Orders are placed at fixed time intervals
- Reasons for using the FOI model:
  - Supplier’s policy may encourage its use
  - May require only periodic checks of inventory level e.g., grocery stores, small shops
  - Grouping orders from the same supplier can produce savings in shipping costs
  - Some circumstances do not lend themselves to continuously monitoring inventory position

Fixed-Interval Benefits

- Tight control of inventory items
- Items from same supplier can be combined and may yield savings in:
  - Ordering
  - Packing
  - Shipping costs
- May be practical when inventories cannot be closely monitored
- Note:

Fixed-Order-Interval Model

- Compare P and Q systems in terms of order quantity, reorder point, order interval (time between order)

<table>
<thead>
<tr>
<th>Order Quantity</th>
<th>Reorder Point</th>
<th>Order Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P system</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- When both demand and lead time are constant, P and Q systems function identically.

Fixed-Interval Benefits

- Tight control of inventory items
- Items from same supplier may yield savings in:
  - Ordering
  - Packing
  - Shipping costs
- May be practical when inventories cannot be closely monitored
Fixed Interval Disadvantages

- Requires a larger safety stock
- Increases carrying cost
- Costs of periodic reviews?

Fixed-order-interval order quantity

Amount to order (target inventory) =

\[ \bar{d} \ast (OI + LT) + Z \ast \sigma_d \ast \sqrt{OI + LT} - A \]

Where

- OI = order (review) interval
- A = inventory on hand at reorder time

FOI Example

a. Given: D=520 units, EOQ=62 units, 99% service level, \( d_x = 9 \) units/week, LT=3 week, A=10
b. Reorder Interval

\[ P = OI = (EOQ/D)(52) = (62/52)(52) = 6.2 \text{ or 6 week} \]
c. Safety stock=
d. Amount to order =

Single Period Model

- **Single period model**: model for ordering of perishables and other items with limited useful lives
- **Shortage cost**: generally the unrealized profits per unit

\[ C_s = \]

- **Excess cost**: difference between purchase cost and salvage value of items left over at the end of a period

\[ C_e = \]

Stocking Levels

Service level

\[ C_s \]

where

- \( C_s \) = shortage cost per unit
- \( C_e \) = excess cost per unit

\[ S_o = \text{Optimum Stocking Quantity} \]

Optimal stocking point =

Single Period Model Example

- Great Farmer’s Market buys organic mixed salad for $2.00 per pound and sells it for $4.20 per pound. At the end of each week, any remaining mixed salad is sold to a producer of canned soup for $0.6 per pound. Weekly demand can be approximately by a normal distribution with a mean of 100 pounds and a standard deviation of 10 pounds. What is the optimal stocking level?
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Areas under the standard normal curve, from $-\infty$ to $+z$

<table>
<thead>
<tr>
<th>$z$</th>
<th>0.00</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.5000</td>
<td>0.5040</td>
<td>0.5079</td>
<td>0.5114</td>
<td>0.5146</td>
<td>0.5177</td>
<td>0.5206</td>
<td>0.5234</td>
<td>0.5261</td>
<td>0.5287</td>
</tr>
<tr>
<td>1.0</td>
<td>0.8413</td>
<td>0.8438</td>
<td>0.8461</td>
<td>0.8485</td>
<td>0.8507</td>
<td>0.8528</td>
<td>0.8548</td>
<td>0.8568</td>
<td>0.8587</td>
<td>0.8607</td>
</tr>
<tr>
<td>2.0</td>
<td>0.9772</td>
<td>0.9783</td>
<td>0.9794</td>
<td>0.9804</td>
<td>0.9813</td>
<td>0.9822</td>
<td>0.9831</td>
<td>0.9839</td>
<td>0.9847</td>
<td>0.9855</td>
</tr>
<tr>
<td>3.0</td>
<td>0.9962</td>
<td>0.9968</td>
<td>0.9972</td>
<td>0.9975</td>
<td>0.9978</td>
<td>0.9981</td>
<td>0.9984</td>
<td>0.9986</td>
<td>0.9988</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

Inventory Management

- Too much inventory
- Tends to hide problems
- Easier to live with problems than to eliminate them
- Costly to maintain
- Wise strategy
- Reduce lot sizes
- Reduce safety stock