Quality Control

Homework Problems: # 1, 2, 4, 5, 7, 9, 13, 14, 22, 24 on pp. 448-453.

Inspection

- Inspection
  - An appraisal activity that compares goods or services to a standard.
  - Inspection issues:
    1. How much to inspect and how often
    2. At what points in the process to inspect
    3. Whether to inspect in a centralized or on-site location
    4. Whether to inspect attributes or variables

How Much to Inspect?

- Raw materials and purchased parts
- Finished products
- Before a costly operation
- Before an irreversible process
- Before a covering process

Examples of Inspection Points

<table>
<thead>
<tr>
<th>Type of business</th>
<th>Inspection points</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fast Food</td>
<td>Cashier, Counter area, Eating area, Kitchen</td>
<td>Accuracy, Appearance, productivity, Cleanliness, Appearance</td>
</tr>
<tr>
<td>Hotel/motel</td>
<td>Parking lot, Accounting, Building, Main desk</td>
<td>Safe, well lighted, Accuracy, timeliness, Appearance, safety, Waiting times</td>
</tr>
<tr>
<td>Supermarket</td>
<td>Cashiers, Deliveries</td>
<td>Accuracy, courtesy, Quality, quantity</td>
</tr>
</tbody>
</table>
Centralized vs. On-Site Inspection

- Effects on cost and level of disruption are a major issue in selecting centralized vs. on-site inspection
  - Centralized
    - Specialized tests that may best be completed in a lab
    - More specialized testing equipment e.g., computer chips
    - More favorable testing environment
  - On-Site
    - Quicker decisions are rendered
    - Avoid introduction of extraneous factors
    - Quality at the source

Statistical Process Control (SPC)

- Quality control seeks
  - Quality of Conformance
    - A product or service conforms to specifications
  - A tool used to help in this process:
    - SPC
      - Statistical evaluation of the output of a process during production or service delivery
      - Helps to decide if a process is “in control” or if corrective action is needed

Process Variability

- Two basic questions concerning variability:
  1. Are the variations random?
     - Process control e.g.,
  2. Given a stable process, is the inherent variability of the process within a range that conforms to performance criteria?
     - Process capability e.g.,

Variation and Control

- Variation
  - Random/Common (cause) variation:
    - Natural variation in the output of a process, created by countless minor factors,
  - Assignable/Special (cause) variation:
    - A variation whose cause can usually be identified

Distribution of a Variable: weight in a box of cereal

What has changed?

Note: the purple (light blue) lines show a change in distribution

Effects of “Assignable Causes” on Process Control

(a) Out of control (assignable causes present)
(b) In control (no assignable causes)
• The essence of statistical process control is to assure that the output of a process is random so that future output will be random as well.

• Sampling Distribution
  • A theoretical distribution that describes the random variability of sample statistics
  • The normal distribution is commonly used for this purpose

• Central Limit Theorem
  • The distribution of sample averages (i.e., sample means) tends to be normal regardless of the shape of the process distribution

• SPC involves periodically taking samples of process output and computing sample statistics:
  • Sample means
  • The number of occurrences of some outcome
  • Sample statistics are used to judge the randomness of process variation

An industrial process that makes 3-foot section of plastic pipe produces pipe with an average inside diameter of 1 (one) inch and a standard deviation of 0.05 inch.

A). If you randomly select one piece of pipe, what is the probability that its inside diameter will exceed 1.02 inches, assuming the population is normal?

B). If you select a random sample of 25 pieces of pipe, what is the probability that the sample mean will exceed 1.02 inches?
Control Chart: The Voice of the Process

- **Control Chart**
  - **Purpose**: to monitor process output to see if it is random
  - A time ordered plot of representative sample statistics obtained from an ongoing process (e.g. sample means), used to distinguish between random and nonrandom variability
  - Upper and lower control limits define the range of acceptable variation

Statistical Control Process

- **The Control Process**
  - Define
  - Measure
  - Compare
  - Evaluate
  - Correct
  - Monitor results
**Quality Control**

**SPC Errors**

- **Type I error**
  - Concluding a process is not in control when it actually is.
  - The probability of rejecting the null hypothesis when the null hypothesis is true.
  - Concluding non-random variation is present when only randomness is present
  - **Manufacturer's Risk**

- **Type II error**
  - Concluding a process is in control when it is not.
  - The probability of failing to reject the null hypothesis when the null hypothesis is false.
  - **Consumer’s Risk**

**Example of Finding Z value**

Q. Find the Z value for 5% \( \alpha \) risk?

**Control Charts for Variables**

**Variables are continuous, generate data that are measured**

- **Mean charts**
  - Used to monitor the central tendency of a process.
  - **X bar charts**
  - **Range charts**
    - Used to monitor the process dispersion
    - **R charts**

**Control Charts for Variables**

- **X bar (mean) charts**
  - Center line is the grand mean (X double double bar)
  - Points are X bars
    \[
    \sigma' = \frac{\sigma}{\sqrt{n}} \\
    \overline{X} = \frac{\sum x_i}{n}
    \]
  - \( UCL = \overline{X} + A_2 \overline{R} \) or \( UCL = \overline{X} + z\sigma' \)
  - \( LCL = \overline{X} - A_2 \overline{R} \) or \( LCL = \overline{X} - z\sigma' \)
  - \( A_2 \) = a control chart factor based on sample size, \( n \) (see Table 10.3)
Range (R) Charts

- Center line is the average Range (R bar)
- Points are R
- $D_3$ and $D_4$ values are tabled according to $n$ (sample size, see Table 10.3)

$$UCL = D_4 \bar{R} \quad LCL = D_3 \bar{R}$$

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Sample 1</th>
<th>Sample 2</th>
<th>Sample 3</th>
<th>Sample 4</th>
<th>R</th>
<th>$ar{x}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5014</td>
<td>0.5022</td>
<td>0.5009</td>
<td>0.5027</td>
<td>R</td>
<td>0.0018</td>
</tr>
<tr>
<td>2</td>
<td>0.5021</td>
<td>0.5041</td>
<td>0.5024</td>
<td>0.5020</td>
<td>R</td>
<td>0.0021</td>
</tr>
<tr>
<td>3</td>
<td>0.5018</td>
<td>0.5026</td>
<td>0.5035</td>
<td>0.5023</td>
<td>R</td>
<td>0.0017</td>
</tr>
<tr>
<td>4</td>
<td>0.5008</td>
<td>0.5034</td>
<td>0.5015</td>
<td>0.5026</td>
<td>R</td>
<td>0.0026</td>
</tr>
<tr>
<td>5</td>
<td>0.5041</td>
<td>0.5056</td>
<td>0.5034</td>
<td>0.5047</td>
<td>R</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

Special Metal Screw

$\overline{X}$-Charts

$$\overline{X} = \frac{\sum x}{n} \quad \bar{x} = 0.5027 \quad A_2 = 0.73$$

$$\text{UCL}_x = \overline{X} + A_2 \bar{R}$$

$$\text{LCL}_x = \overline{X} - A_2 \bar{R}$$

Table 10.3 Factors for 3-sigma control limits for Mean and Range charts

<table>
<thead>
<tr>
<th>$n$</th>
<th>$A_2$</th>
<th>$B_4$</th>
<th>$B_6$</th>
<th>$B_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.48</td>
<td>0.23</td>
<td>0.09</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>0.46</td>
<td>0.22</td>
<td>0.08</td>
<td>0.00</td>
</tr>
<tr>
<td>4</td>
<td>0.45</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>0.44</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>6</td>
<td>0.43</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>7</td>
<td>0.43</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>8</td>
<td>0.43</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
<td>0.43</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
<tr>
<td>10</td>
<td>0.43</td>
<td>0.21</td>
<td>0.07</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Conclusion?
Range Charts Results

- Special Metal Screw

Control Charts

\[ R = 0.0021 \]
\[ D_4 = 2.282 \]
\[ D_3 = 0 \]

\[ \text{UCL}_R = D_4 \bar{R} \]
\[ \text{LCL}_R = D_3 \bar{R} \]

\[ \text{UCL}_x = \]
\[ \text{LCL}_x = \]

Mean and Range Charts

x-Chart

\[ \text{R-chart} \]

\[ \text{sunny Dale Bank} \]
\[ \bar{x} = 5.0 \text{ minutes} \]
\[ \sigma = 1.5 \text{ minutes} \]
\[ n = 6 \text{ customers} \]
\[ z = 1.96 \]

\[ \text{UCL}_x = \]
\[ \text{LCL}_x = \]

Using Mean and Range Charts

- To determine initial control limits:
  - Obtain 20 to 25 samples
  - Compute appropriate sample statistics e.g., mean, range...
  - Establish preliminary control limits
  - Determine if any points fall outside the control limits
    - If you find no out-of-control signals, assume the process is in control. (you can use these control limits)
    - If you find an out-of-control signal, search for and correct the assignable cause of variation. Then resume the process and collect another set of observations on which control limits can be based.
  - Plot the data on the control chart and check for out-of-control signals
**Control Chart for Attributes**

Attributes are discrete, generate data that are counted.

- **P-Chart:** Control chart used to monitor the proportion of defectives in a process

- **C-Chart:** Control chart used to monitor the number of defects per unit

**Use of p-Charts**

- When observations can be placed into two categories.
  - Good or bad
  - Pass or fail
  - Operate or don’t operate
- When the data consists of multiple samples of several observations each

**P-Charts**

\[
\hat{p} = \frac{\text{Total number of defectives}}{\text{Total number of observations}}
\]

\[
\sigma_p = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}
\]

\[
\text{UCL}_p = \hat{p} + z(\sigma_p)
\]

\[
\text{LCL}_p = \hat{p} - z(\sigma_p)
\]

If \( \hat{p} \) is unknown, it can be estimated from samples. \( \hat{p} \) replaces \( p \), \( \sigma_p \) replaces \( \sigma \).

**P-Charts Example**

**Hometown Bank (using \( Z=3 \))**

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Wrong Account Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>19</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>11</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>147</strong></td>
</tr>
</tbody>
</table>

Given \( n = 2500 \)

**P-Charts Plots**

**C-Charts**

- C-charts: used to count defects in a constant sample size

\[
\bar{c} = \frac{\sum_{i=1}^{n} c}{m} = \text{centerline}
\]

\[
\text{UCL} = \bar{c} + z\sqrt{\bar{c}}
\]

\[
\text{LCL} = \bar{c} - z\sqrt{\bar{c}}
\]

Conclusion?
C-Charts Example

- A Highway intersection averages 3 accidents per month. Last month 7 accidents occurred. Has something changed at the intersection? Use Z=3.
  Q. Does this appear to be a quality control problem?

Use of c-Charts

- Use only when the number of occurrences per unit of measure can be counted; non-occurrences cannot be counted.
  - Scratches, chips, dents, or errors per item
  - Cracks or faults per unit of distance
  - Breaks or Tears per unit of area
  - Bacteria or pollutants per unit of volume
  - Calls, complaints, failures per unit of time

Run Tests

- Even if a process appears to be in control, the data may still not reflect a random process
- Analysts often supplement control charts with a run test
  - Run test
  - A test for patterns in a sequence
  - Run
  - Sequence of observations with a certain characteristic

Nonrandom Patterns in Control charts

- Trend
- Cycles
- Bias
- Mean shift
- Too much dispersion

Counting Runs

Q. How to find the median for an odd or even number of observations?
Counting Runs

Figure 10.12
Median tests
Counting Above/Below Median Runs (7 runs)
B A A B A B B B A A B

Figure 10.13
Up/down tests
Counting Up/Down Runs (8 runs)
U D U D U D U D U

First observation

Run Tests Example

- The number of defective items per sample for 11 samples is shown below. Determine if random patterns are present in the sequence using both median and Up-down tests. Use Z=2.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Number of defectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>22</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
</tr>
<tr>
<td>3</td>
<td>19</td>
</tr>
<tr>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
</tr>
<tr>
<td>6</td>
<td>21</td>
</tr>
<tr>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>8</td>
<td>23</td>
</tr>
<tr>
<td>9</td>
<td>24</td>
</tr>
</tbody>
</table>

Run Tests Formula (Table 10.4)

<table>
<thead>
<tr>
<th>RUN TESTS</th>
<th>NUMBER OF RUNS</th>
<th>N</th>
<th>Standard Deviation</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>( r )</td>
<td>( \frac{n-1}{2} \times 1 )</td>
<td>( \frac{r}{\sqrt{n-1}} )</td>
<td>( \frac{\sqrt{n-1}}{2} )</td>
</tr>
<tr>
<td>Up/down</td>
<td>( r )</td>
<td>( \frac{n-1}{2} \times 3 )</td>
<td>( \frac{n-1}{3\sqrt{n-1}} )</td>
<td>( \frac{n-1}{3\sqrt{n-1}} )</td>
</tr>
</tbody>
</table>

N: the # of samples

Process Capability

- Specifications or tolerances
  - Range of acceptable values established by engineering design or customer requirements
- Process variability
  - Natural variability in a process
- Process capability
  - Process variability relative to specification see examples

- One of the unique properties of a normal distribution is that 99.74% of the observations will fall within 3 standard deviation (\( \sigma \)) from the mean (\( \mu \)). Thus, we expect a process that is in control to produce the majority of the output (99.74%) between \( \mu - 3\sigma \) and \( \mu + 3\sigma \), where \( \mu \) is the process mean. That is, the natural tolerance limits of the process are \( \mu \pm 3\sigma \), and thus we use \( 6\sigma \) as a measure of process capability.

Process Capability Ratio

Process capability ratio, \( C_p = \frac{\text{specification width}}{\text{process width}} \)

\[ C_p = \frac{\text{Upper specification} - \text{lower specification}}{6\sigma} \]

- Note that a \( C_p \) value less than 1.0 indicates that the process is not capable of meeting specifications; a value \( \geq 1.0 \) corresponds to a process that is capable of meeting specification.
- A \( C_p \) value of 1.0 implies that the firm is producing 3\( \sigma \) quality (0.26% defects) and that the process is consistently producing outputs within specifications even though some defects are generated. Pretty good?
A $C_p$ value of 1.0 implies that the firm is producing 3σ quality (0.26% defects). Fairly decent indeed, but

- 7 lost pieces of mail every day.
- 7 wrong drug prescriptions each year
- 7 incorrect surgical operations each year.

$C_p = 0.66 \Rightarrow 2\sigma$ quality =

$C_p = 1.0 \Rightarrow 3\sigma$ quality =

$C_p = 1.33 \Rightarrow 4\sigma$ quality =

$C_p = 2.0 \Rightarrow 6\sigma$ quality =

Note that unless the process mean is centered between the upper and lower specifications, the $C_p$ ratio can be misleading.

Process Capability Index

Process Capability Index, $C_{pk} = \min (C_{pu}, C_{pl})$

Where,

$C_{pu} = \frac{\text{Upper specification} - \mu}{3\sigma}$

$C_{pl} = \frac{\mu - \text{Lower specification}}{3\sigma}$

$\mu = \text{the process average/mean}$

Process Capability Ratio & Index Example

- The intensive care unit lab process has an average turnaround time of 26.1 minutes and a standard deviation of 1.2 minutes. The target (nominal) value for this service is 25 minutes with an upper specification limit of 30 minutes and a lower specification limit of 20 minutes. Management wants to have four-sigma performance for the lab. Is the lab capable of this level of performance?

$C_p =$

$C_{pk} =$

Product/service Reliability

- Series System

$P_1 \rightarrow P_2 \rightarrow \cdots \rightarrow P_n$

$P_i = \text{probability that component } i \text{ will function}$

Reliability of the product/system = $P_1 \times P_2 \times \cdots \times P_n$

Example:

- Parallel System

$P_1 \rightarrow P_2 \rightarrow \cdots \rightarrow P_n$

Reliability = $1 - [1-P_1] \times [1-P_2] \times \cdots \times [1-P_n]$

Example: